International Journal of Scientific & Engineering Research, Volume 11, Issue 3, March-2020
ISSN 2229-5518

Improved Oil and Gas Production Decline Curve Fit Using Rate – Time Linearized Form of Hyperbolic Decline

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Abstract— Production forecasting using Decline curve is common but limited in accuracy. The Exponential, Harmonic and Hyperbolic declines originated by J.J. Arps are three main decline profile types widely used in the industry. Exponential and Harmonic curves are more common than the hyperbolic ones largely because of the ease of fitting oil and gas production profiles with the earlier curves. However, Hyperbolic is the principal Decline curve equation while Exponential and Harmonic, which rarely occur, are special cases of the general Arps' Decline curve. The difficulty in Hyperbolic fit is in the prediction of the Decline exponent.

The methods used to fit hyperbolic curves today over oil and gas decline profiles are time consuming computer programs requiring multiple iterations. These fits are not exact hyperbolic representations because of the numerical truncations that are inherent in the programs. Numerical approximations were used because there were no known linear forms that fitted the hyperbolic decline equation from real production data. This paper shows a rate time linearized form of Hyperbolic Decline. The novel form was found to be a better representation of the hyperbolic form than those from other known hyperbolic programs.

It should be noted that the linear forms presented here only reproduces the Hyperbolic Decline form but cannot account for the decline pattern exhibited by wells due to drastic changes in reservoir fluid or rock properties like Gas-oil-ratio, water cut, porosity, permeability and changes caused by introduction of Artificial Lift or enhanced recovery scheme. The rate-rate-derivative time linear form only worked for a smooth data with a defined hyperbolic decline pattern while the decline constant harmonization method is used to fit any other data that cannot be fit with the former.

Index Terms— Decline curve analysis, Hyperbolic decline pattern, Linear rate-rate-derivative time plots, Natural reservoir pressure, Performance history, Production forecast, Reserve estimation.

1 INTRODUCTION

Oil and gas production rates usually decline as a function of time as natural reservoir pressure diminishes. Fitting a line through the performance history and assuming this same line trends similarly into the future forms the basis for the decline curve analysis (DCA) concept. Decline Curve Analysis is not an exact science, but a way to mimic production trend as production rate decreases. The usefulness of a decline curve tool lies in its ability to fit the decline trend, forecast with some measure of accuracy, the future trend and estimate reserves with negligible error margin. The basic assumption in this procedure is that whatever causes controlled the trend of a curve in the past will continue to govern its trend in the future in a uniform manner.

Arps (1945 and1956) established the foundation for decline curve analysis by collecting these ideas into a comprehensive set of line equations defining exponential, hyperbolic and harmonic curves.
Brons (1963) and Fetkovich (1983) demonstrated that the DCA is more than just an empirical curve fit by applying the constant pressure solution to the diffusivity equation to show that the exponential decline curve actually reflects single phase, incompressible fluid production from a closed reservoir.
M.J Fetkovich, (1980 and 1983), developed set of type curves to enhance application of DCA. He established exact methods of fitting Hyperbolic Decline curves by type curve matching using discreet points of decline exponents. The major limitation is that it does not determine the unique decline exponent for any given decline trend. It is however presently relied upon as one of the best methods for estimating decline exponent.

The general aim of DCA is to model production history with the equation of a straight line. However, there is no straight-line form of the hyperbolic decline equation documented in the literatures of DCA. This point is further underscored by the fact that there are a lot of programs and subroutines that fit hyperbolic decline profiles, and none exists for either exponential or harmonic decline profiles.

The aim of this paper is to establish linear forms of Hyperbolic decline that can be used to fit decline trend. This will simplify the hyperbolic decline fitting, reduce computer time, and eliminate errors due to approximations. Furthermore, it would provide a unique linear form whose extrapolate is intrinsic to the fit and a perfect representation of hyperbolic profile over the decline trend.

It should be noted that the transformed linear forms presented in this paper are appearing for the first time in the literatures of Engineering and Science at large.

International Journal of Scientific & Engineering Research, Volume 11, Issue 3, March-2020
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2 DECLINE CURVE ANALYSIS

2.1 ARPS DECLINE CURVE

A great number of studies on production data are based on J. J. Arps (1945) decline curve analysis which presents the relationship between production and time during pseudo-steady state period and is expressed as:

$$q = \frac{q_i}{(1+bDt)^{\frac{1}{b}}} \quad (1)$$

Where the constants:
q_i = Initial Production rate
D = Decline constant
b = Decline exponent

Arps (1956), applied the equation of a hyperbola to define three general equations (exponential, hyperbolic and harmonic) to model production declines as follows

1. Exponential (Constant percentage) Decline (b = 0)

$$q = q_i e^{-Dt} \quad (1a)$$

2. Hyperbolic Decline (0<b<1) :

$$q = \frac{q_i}{(1+bDt)^{\frac{1}{b}}} \quad (1)$$

3. Harmonic Decline (b=1) :

$$q = \frac{q_i}{1+Dt} \quad (1b)$$

It is easily observed from either binomial approximation of equations (1) and (1b) and maclaurin series approximation that when the product Dt is far less than 1,

$$q = q_i(1-Dt) \quad (1c)$$

In order to locate a hyperbola in space, the following parameters must be known:

The starting point on the y axis,

- Initial rate, q_i;
- Initial decline rate, D_i; and
- The degree of curvature of the line, b.

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2.2 LINEAR FORMS OF ARPS DECLINE EQUATIONS

The linear forms used to evaluate the constants in Arps Decline equation are:

(i) Exponential Decline:

$$\ln q = \ln q_i - Dt \quad (2)$$

(ii) Harmonic Decline

$$1/q = 1/q_i(1+Dt) \quad (3)$$

Equations (2) and (3) are easily inferred from the equations (1a) & (1b) respectively.

2.3 LINEARIZATION OF THE ARPS HYPERBOLIC DECLINE EQUATIONS

This paper demonstrates that the linear form of hyperbolic decline equation could be obtained by combination of a third parameter to the hyperbolic decline rate equation (equation (1)).

(i) The linear form of the Hyperbolic Decline presented is a rate-rate derivative-time function – which is an equation involving Instantaneous production rate decrease with time, Production rate and time.

(ii) The rate-rate derivative-time function method has limited application. It can only be used when the Production rate has shown the Decline pattern.

(i) RATE – RATE DERIVATIVE -TIME METHOD

Predominantly Water driven reservoirs produce at a rate which when divided with its derivative (instantaneous decline rate with time) is in partial linear variation with time. These reservoirs are produced by production fluid expansion and displacement by another fluid.

Hyperbolic decline occurs when the decline rate is no longer constant. Hyperbolic decline-curve equations estimate a longer production life of the well when compared to the exponential decline equation. Hyperbolic decline curve corresponds to a value of b in the range 0 <b<1.

- The hyperbolic Decline equation, from equation (1) is given by:

$$q = q_i/(1+bD_i t)^{1/b} \quad (1a)$$

Where,
q = current production rate,
q_i = initial production rate (start of production)
D_i = initial nominal decline rate at $t = 0$ (defined at the same time as the initial production rate)
t = cumulative time since start of production
b = hyperbolic decline constant $(0 < b < 1)$

A graphical representation of the hyperbolic decline equation Eq. (1a) encompasses the whole range of conditions from exponential decline (b = 0) to harmonic decline (b = 1), where each value of initial rate (q_i), initial nominal decline rate (D_i), and decline exponent (b) produces its own unique curve.
The rate of decrease in production with time, is expressed as:

$$dq/dt = -(q_i/b) * (1+bDt)^{-1/b-1} * bD \quad (4)$$

Introducing a rate-time function, τ, defined as rate per instantaneous rate change with time:

$$\tau = -q/(dq/dt) = 1/D + bt \quad (5)$$

The transformed straight-line rate time function is then given by

$$\tau = 1/D + bt \quad (6)$$

Where $\tau =$ the modified rate time function

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A plot of τ against t gives a straight line with gradient = b and intercept $=1/D$, the inverse of Decline constant, D as shown in fig. 1. Below

Intercept $=1/D$ (6a)

$Slope = b$ (6b)

This Implies that,

$D = 1/intercept$ (6ai)

And

$b = Slope$ (6bi)

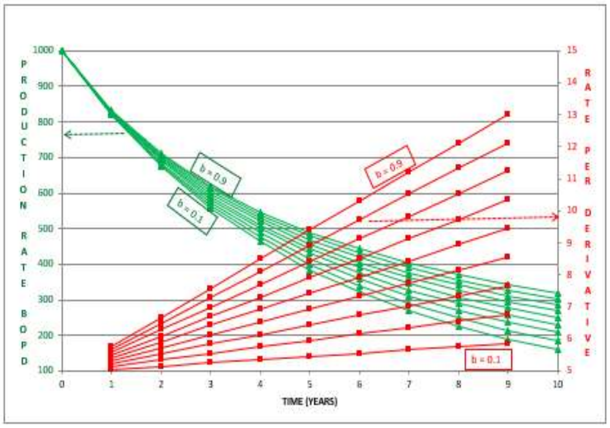

Fig. 1. Rate-Rate Derivative - Linear Transform Plot derived from a family of hyperbolic decline curves

SCATTERED DATA

Real production data are not often smooth. This has been explained above by Feilong et al. (2009). It is usually popular practice to smoothen these data by a form of averaging method and picking a set of data representative of the entire data by "eye balling" method. All these methods erode the decline curve properties and affect the final decline curve fit.

(i) Production rate-time data smoothening

Arithmetic Average Method –This is the simplest method of averaging rate and as the name implies, it is the Arithmetic mean of the rates within the time frame of interest. Hence represented in the form,

$$q_r = (q_{r1} + q_{r2} + q_{r3} \dots + q_{rn})/n \qquad (7)$$

This is also the least accurate of the three smoothening methods discussed.

Time-weighted rate Average Method –

This involves the use of rate validity time (time a rate was prevalent in the data) to calculate the rate representative over a given time frame and it is simply,

$$q_r = (q_{r1}\Delta t_1 + q_{r2}\Delta t_2 + \dots + q_{rn}\Delta t_n)/(\Delta t_1 + \Delta t_2 + \dots + \Delta t_n) \qquad (8)$$

Proposed rate-time Method

This involves fitting a decline curve straight line approximation in equation (1c) below on production rates within a short period. This straight-line approximation fits all decline (exponential, harmonic and hyperbolic) profile. The decline rate is not a constant but changes with time so that:

$$q = q_{i.n}\left(1 - D_{i.n}(t - t_n)\right) \qquad (9)$$

It should be noted that this equation improves as t approaches t_n and becomes perfect when t = t_n .These imply that the $q_{i.n}$ (the intercept of the equation above) is the best representation of the periodic flow rate in the time t_n of the decline profile.

Another quality to be used to validate the set of data is that the periodic decline, $D_{i.n}$, will gradually reduce with periodic succession. This is further discussed in the next section.

Decline Constant Harmonization:

The derived linear forms above are very sensitive to scattered data and often outputs inconsistent profile (Numerically dispersion) that deviate from real data when extrapolated. This occurs when the periodic decline constant is not in a sequential decreasing order. It is therefore necessary to harmonize these periodic decline constants and then regenerate a new flow rate versus time before fitting the profile. The formulas for this harmonization are all derived below.

The hyperbolic Decline equation, Eq. (1) if differentiated gives:

$$dq/dt = -q_i D(1 + bDt)^{-(b+1/b)} \qquad (10)$$

Matching the rate derivative of this equation (9) and equation (10) it is deduced that at any stage of the hyperbolic decline, the instantaneous decline constant is given by,

$$q_{i,n}D_{i,n} = -q_i D(1 + bDt)^{-(b+1/b)} \qquad (11)$$

Adopting a stepwise time incremental value, t_p, such that the time series below are derived:

$$0, t_p, 2t_p, 3t_p, 4t_p, \quad . \; . \; . \; . \quad , mt_p \qquad \text{(Series (1))}$$

Recalling also that,

$$q_{i,n}(t_p) = q(t_p) = q_i/(1 + bDt_p)^{1/b} \qquad (12)$$

Substituting into Eqn. (11) and earanging gives:

$$D_{i,n}(t_p) = D(1 + bDt_p)^{-1} \qquad (13)$$

Generating instantaneous decline constants for every value of n gives the series below

$$D, D(1 + bDt_p)^{-1}, D(1 + 2bDt_p)^{-1}, D(1 + 3bDt_p)^{-1}, \dots\dots, D(1 + mbDt_p)^{-1} \qquad \text{(Series (2))}$$

The recurring factor in the above series is

$$f_{rn} = (1 + nbDt_p)^{-1} \qquad (14)$$

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If t_p is to be chosen such that
$t_p \ll 1/D$

(Condition (1))

Since the Arp's decline exponent b, is within the range: 0<b<1, then by linear approximation to binomial theorem, the right-hand side of equation (14) becomes

$$(1+nbDt_p)^{-1} \approx (1+bDt_p)^{-n} = \left[(1+bDt_p)^{-1}\right]^n$$
(15)

Hence (series 2) is reduced to
$$D, D(1+bDt_p)^{-1}, D\left((1+bDt_p)^{-1}\right)^2, D\left((1+bDt_p)^{-1}\right)^3, \ldots\ldots, D\left((1+bDt_p)^{-1}\right)^n$$
(Series (3))

Series (3) is a geometrical series reproducible by the equation below

$$D_{i,n}(nt_p) = D\left[(1+bDt_p)^{-1}\right]^n \quad (16)$$

$$\Rightarrow log\left(D_{i,n}(nt_p)\right) = log\, D - n\, log(1+bDt_p) \quad (17)$$

This can be simplified further from condition (1) & (2), above and below respectively and the log approximations below.

Therefore a plot of $log(D_{i,n}(nt_p))$ versus n will give a straight line

$x << 1$, (Condition (2))

$ln(1+x) \approx x$ (Logarithmic approximation (1))

$log(1+x) = 0.434294 * ln(1+x) \approx 0.434294 * x$
(Logarithmic approximation (2))

$$\Rightarrow log\left(D_{in}(nt_p)\right) = log\, D - n\, log(1+bDt_p) \approx log\, D - 0.434294 * n * bDt_p \quad (18a)$$

$$\Rightarrow \boldsymbol{log\left(D_{in}(nt_p)\right) = log\, D - n * 0.434294 * bDt_p} \quad (18b)$$

Again a plot of $log\left(D_{in}(nt_p)\right)$ versus n will give a straight line with intercept as $log\, D$ and slope of $-0.434294\, bDt_p$. The above is a simplified linear form of hyperbolic decline.

TABLE 1
TEST DATA (RATE- RATE DERIVATIVE METHODOLOGY)

t(years)	t(days)	q(BOPD)	Rate Derivative (BOPD/YEAR)	Rate/rate Derivative (PER YEAR)	q (trended) BOPD
0	0	1000			1000
0.125	45.65625	975.442151	192.8882011	5.05703379	975.4345652
0.25	91.3125	951.7779497	185.9103749	5.119552635	951.7633844
0.375	136.96875	928.9645572	179.2651148	5.182071025	928.9435726
0.5	182.625	906.961671	172.9328409	5.244588977	906.9347849
0.625	228.28125	885.731347	166.8953404	5.307106506	885.6990385
0.75	273.9375	865.2378359	161.1356579	5.369623627	865.2005483
0.875	319.59375	845.4474325	155.6379948	5.432140355	845.4055761
1	365.25	826.3283372			826.2822922
1.25	456.5625	789.9856422			789.9322507
1.5	547.875	755.988846			755.9293207
1.75	639.1875	724.1403069			724.0756912
2	730.5	694.2627671			694.19396
2.25	821.8125	666.1968813			666.1246578
2.5	913.125	639.7990873			639.7241163
2.75	1004.4375	614.9397669			614.8626257
3	1095.75	591.5016513			591.4228385
3.25	1187.0625	569.3784361			569.2983821
3.5	1278.375	548.4735732			548.3926496
3.75	1369.6875	528.6992162			528.6177437
4	1461	509.9752956			509.8935505
4.25	1552.3125	492.2287073			492.1469276
4.5	1643.625	475.3925977			475.310988
4.75	1734.9375	459.4057333			459.324469
5	1826.25	444.2119431			444.1311745
5.25	1917.5625	429.7596247			429.6794799
5.5	2008.875	416.0013053			415.9218929
5.75	2100.1875	402.8932514			402.8146635
6	2191.5	390.395121			390.317435
6.25	2282.8125	378.4696529			378.392933
6.5	2374.125	367.0823882			367.0066877
6.75	2465.4375	356.2014216			356.1267839
7	2556.75	345.7971774			345.7236373
7.25	2648.0625	335.8422084			335.7697933
7.5	2739.375	326.3110146			326.2397452
7.75	2830.6875	317.17988			317.1097714
8	2922	308.4267245			308.3577869
8.25	3013.3125	300.0309706			299.9632098
8.5	3104.625	291.9734226			291.9068406
8.75	3195.9375	284.2361559			284.1707516
9	3287.25	276.8024184			276.7381876
9.25	3378.5625	269.6565387			269.5934749
9.5	3469.875	262.783844			262.7219388
9.75	3561.1875	256.1705847			256.1098277
10	3652.5	249.8038655			249.7442447

TABLE 2
TEST DATA FIT (RATE- RATE DERIVATIVE METHOD SOLUTION)

DECLINE PARAMETERS	FORMULA	VALUE	SOURCE
Decline constant (/Year)	D= 1/INTERCEPT=	0.200220242	Derived from the graph
Decline exponent	b= GRADIENT =	0.5001	Derived from the graph
Initial Rate (BOPD)	q_i = INITIAL RATE	1000	Derived from the Data

Initial rate was taken directly from data as there was a perfect linear fit (R^2=1)

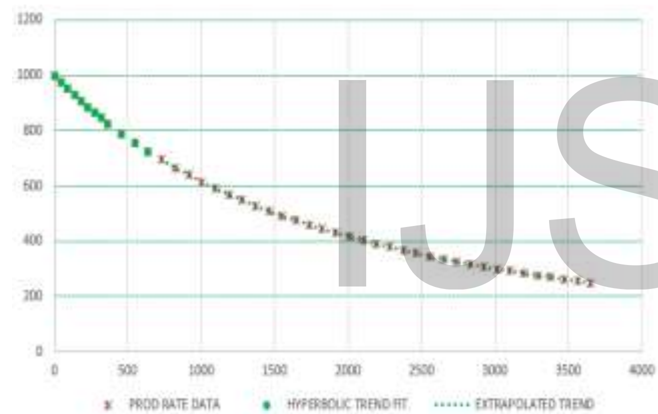
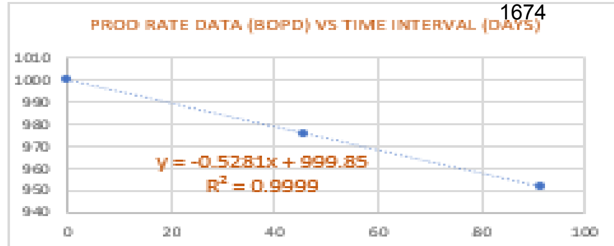
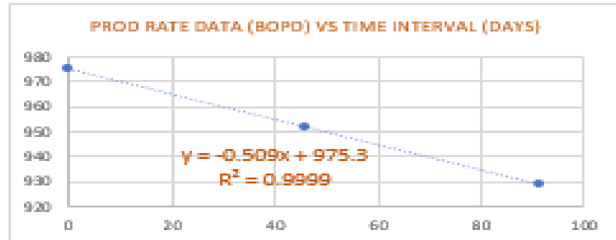
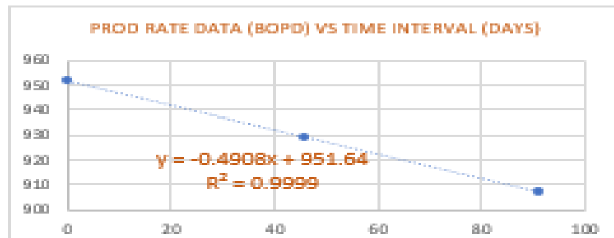
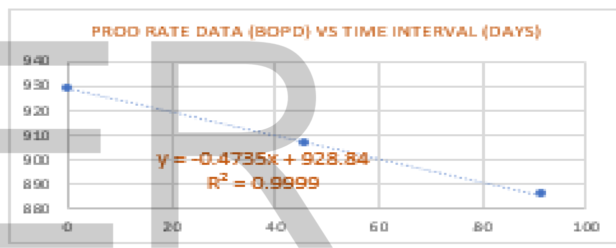


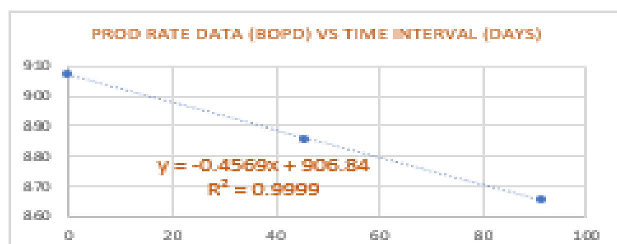
Fig. 3. Production rate (BOPD) versus time (Days) hyperbolic fit for Table 1. Data and trend extrapolated for 10 years











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ISSN 2229-5518

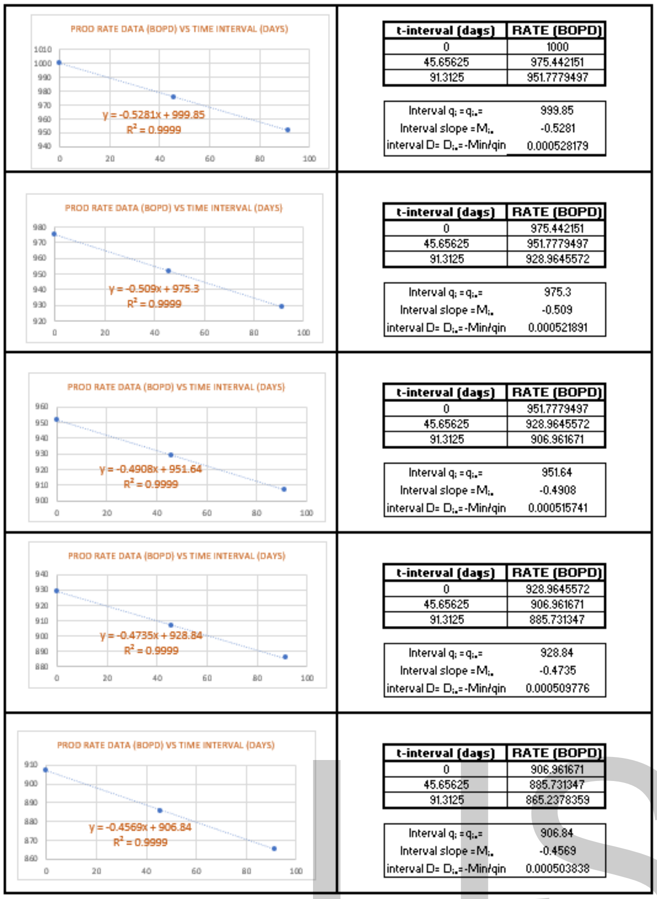

Fig. 4. Estimation of interval initial rate and decline constant

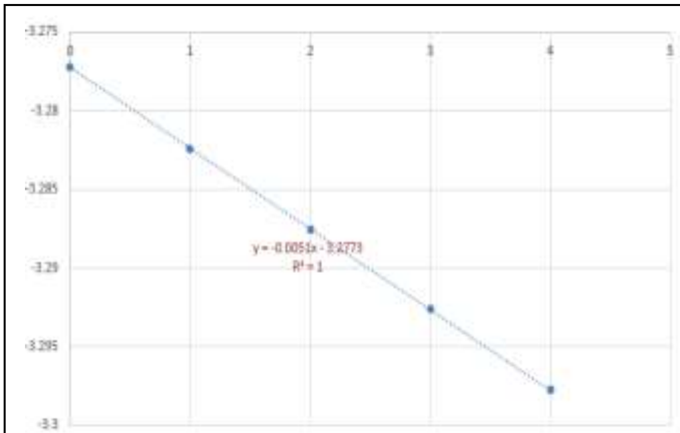

Fig. 5. Log ($D_{in}(nt_p)$) versus n **for test data** to determine the decline constant and decline exponent

TABLE 4
TEST DATA FIT (DECLINE CONSTANT HARMONIZATION SOLUTION)

DECLINE PARAMETERS	FORMULA	VALUE	SOURCE
Decline constant (/day)	Decline constant harmonization	0.000528	Derived from the graph
Decline exponent	Decline constant harmonization	0.489532	Derived from the graph
Initial Rate (BOPD)	Interval initial rate regression	998.164	Derived from regression

Initial rate derived from regression of interval initial rates used in the hyperbolic fit

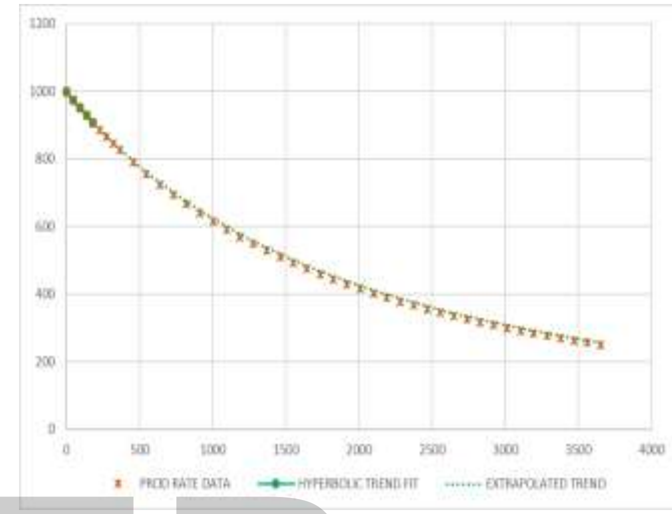

Fig. 6. Production Rate (BOPD) Versus Time (Days) for Decline Constant Harmonisation on the Test Data and extrapolated for 10 years

TABLE 5
PRODUCTION DATA (DECLINE CONSTANT HARMONIZATION METHODOLOGY)

t(months)	q(MBOPM)	q_{in}(MBOPM)	q (trended) MBOPM
0			415.020637
1	400	399.83	398.9852501
2	384	383.83	384.0119717
3	369	369.83	370.0027782
4	355		356.871108
5	342		344.5402537
6	330		332.9420158
7	318		322.0155689
8	307		311.7065034
9	296		301.9660117
10	286		292.750194
11	276		284.0194641
12	267		275.7380389

Initial rate was derived by trending the hyperbolic fit to time zero

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TABLE 6
PRODUCTION DATA FIT (DECLINE CONSTANT HARMONIZATION SOLUTION)

DECLINE PARAMETERS	FORMULA	VALUE	SOURCE
Decline constant (/Month)	Decline constant harmonization	0.000528	Derived from the graph
Decline exponent	Decline constant harmonization	0.489532	Derived from the graph
Initial Rate (MBOPM)	Interval initial rate regression	998.164	Derived from regression

Initial rate got from regression of interval initial rates used in hyperbolic fit

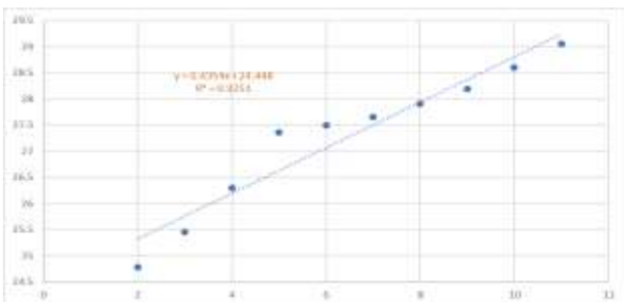
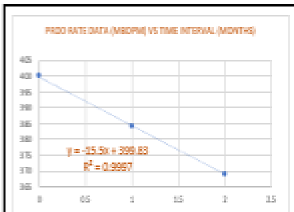
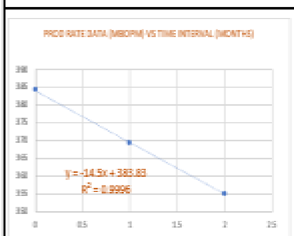


Fig. 7. Rate/Rate Derivative (Months) Versus Time (Months) and plot showed that hyperbolic trend not established from trend and hence Decline Constant Harmonization method should be used instead.



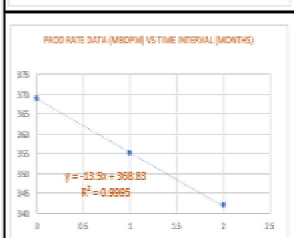
t-interval (months)	RATE (MBOPM)
0	400
1	384
2	369

Interval $q_i = q_{i,n} =$	399.83
Interval slope $= M_n$	-15.5
interval D= $D_{i,n} = -M_n/q_{in}$	0.038766



t-interval (months)	RATE (MBOPM)
0	384
1	369
2	355

Interval $q_i = q_{i,n} =$	383.83
Interval slope $= M_n$	-14.5
interval D= $D_{i,n} = -M_n/q_{in}$	0.037777



t-interval (months)	RATE (MBOPM)
0	369
1	355
2	342

Interval $q_i = q_{i,n} =$	368.83
Interval slope $= M_n$	-13.5
interval D= $D_{i,n} = -M_n/q_{in}$	0.036603

Fig. 8. Estimation of interval initial rate and decline constant.

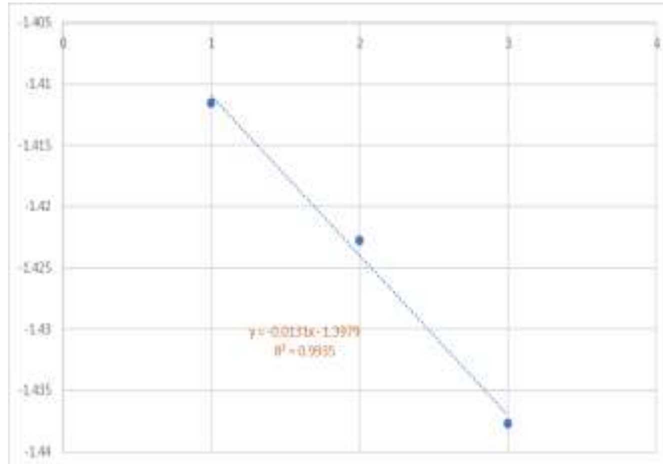


Fig. 9. Log ($D_{in}(nt_p)$) versus n **for production data** to determine the decline constant and decline exponent

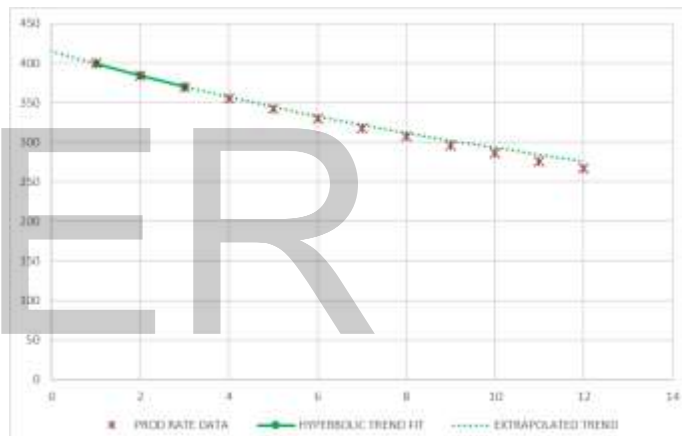


Fig. 10. Production Rate (BOPD) Versus Time (Days) for Decline Constant Harmonisation on the Test Data and extrapolated

Rate-rate derivative - time fit

Rate-rate derivative time method of fitting hyperbolic is simplified and more accurate when a scattered data is smoothened. The main source of the error is the production rate data and this is further amplified when a derivative of rate with respect to time is obtained from such data. However, this is minimized when the rate-time data is smoothened as discussed above.

Rate derivative evaluation

The derivative of rate with respect to time is calculated from the smoothened data. The numerical difference equation can be used to calculate these from rate and time. The most common of them is the forward difference but backward difference equations can also be used. Backward and forward difference

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ISSN 2229-5518

equations have no problem at the first and last node respectively. Central difference, being the least common due to its complex methodology, is the most accurate, although extreme (first and last) nodes cannot be evaluated.

Decline Constant Harmonization Method fit

The Decline constant Harmonization Method is best used for scattered data after smoothening and form the analysis the fit error is minimal within the recommended limit. The smaller the time interval the lower the error. However, it is important to note that the range of data to be fit must be in such that the product bDt is less than or equal 0.1. The best way to determine this is to assume D to be the first value of decline constant during data smoothening and multiply with the absolute time. Decline Exponent b, for Arps decline constant is always between 0 and 1 hence the Dt product is good enough to select the data range for the fit.

Initial rate evaluation for scattered data

The derived linear form for rate-derivative-time and the decline constant harmonization fits does not allow determination of initial rate from the fitting process. It is normally compelling to use the first rate as initial rate or derive this from the first periodic fit in the smoothening linear approximation of rate and time. However, none of these is entire data representative and should not be practiced for the purpose engineering accuracy.

The inability of the derived equations above to determine initial production rate is because the methodology that was used to derive the linear functions eliminated the initial rate in the final equation - even if there was, one could not solve three unknowns with two equations. The best meaningful way to determine initial rate, q_i used a minimizing error analysis (with reference to the smoothened data) in a direct calculation of the periodic flow rates. This was achieved by using a goal seek function in the excel spread sheet that calculated the different periodic flow rates with zero (or approximately zero) total deviation. All these are summarized on the flow chart in Appendix.

It is also important to note that at this region of solution, the product of decline exponent, decline constant and time of production must not be greater than 0.1 for this solution to be valid and at this point it may be good to start a first guess with exponentially determined initial rate:
From Eq. (1) the following is valid:

$$ln\, q = ln\, q_i - (1/b) ln(1 + bDt) \qquad (19)$$

For the period within which the product bDt will be less than 0.1, the log approximation (1) at condition (2) will be prevalent hence applying these conditions to equation (19), to give:

$$ln\, q = ln\, q_i - Dt \qquad (20)$$

The above equation is same as the exponential linear form in Eq. (2).

CONCLUSION AND RECOMMENDATION

The Rate-Rate derivative-Time linear form of the hyperbolic decline curve that was presented in the paper reproduces the Arps hyperbolic decline profile. This is only usable for non-scattered data because of the need to calculate, numerically, the derivative of the production rate with respect to time which, will be inaccurate for scattered data.

Again, this method is limited because the rate of change of production rate with time is not a measurable quantity from the field.The usefulness of this method is then limited to fitting of hyperbolic curve whose trend is already defined from the data and the error of fit grows with data dispersion.

A modification of the rate time equation was made to address these above challenges especially fitting the scattered data. This was done by fitting the estimated stepwise decline constant from the production -time data after harmonizing the decline constant. The decline constant and decline exponent derived from this method, which are deterministic, are representative of the data.

The demerit of this method is the inability of the equations to predict the initial rate that is representative of the entire scattered data. This was largely because the number of variables to be determined (by Eq. (3)) are more than the resulting systems of equations (Eq. (2)) which is not evaluable because of insufficient data. The initial rate evaluation was addressed with an iterative methodology which ensures minimal deviation errors from the measuredrate-time data.

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International Journal of Scientific & Engineering Research, Volume 11, Issue 3, March-2020
ISSN 2229-5518

APPENDIX